Journal of Statistical Physics, Vol. 103, Nos. 3/4, 2001

At the Dawn of a Unifying Theory

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Received June 27, 2000

I present a brief summary of the recent advancements on the unification of abelian quantum Hall states that covers the effects of disorder as well as the Coulomb interactions. I also present some exact results on the CP^{N-1} model at large N. In contrast to the previous expectations, this theory shows all the basic features of the quantum Hall effect, including the massless bulk excitations at $\theta = \pi$.

KEY WORDS: Abelian quantum Hall states; instanton vacuum; Chern Simons gauge theory; unification; CP^{N-1} theory.

1. INTRODUCTION

The *instanton vacuum* was originally introduced in quantum Hall physics by the author[(1)] while studying a fundamental problem in the scaling theory of Anderson localization (non-linear σ model). Levine, Libby and the author[(2)] subsequently elaborated on the highly non-trivial and likely consequences of having a θ term in replica field theory by capitalizing on the existing similarities between the theory and QCD.

Since then, topological concepts in replica field theory have set the stage for several interesting ideas and new developments. They have led, amongst other things, to the previously unrecognized concept of θ *renormalization* by instantons.[(3, 4)] Additionally, they have also facilitated the prediction of a *quantum phase transition* between adjacent quantum Hall plateaus.[(5)] Following the extended experimental work by H. P. Wei *et al.*,[(6, 7)] the details of the plateau transitions in the quantum Hall regime (i.e., the numerical value of the critical exponents) have been a topic of ongoing research, for many years.

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0022-4715/01/0500-0569$19.50/0

However, although the subject matter has been pursued by various streams of ongoing research, several fundamental aspects of the quantum Hall effect have remained unexplained. For example, it has not been clear, for a long time, whether the replica field theory indeed predicts an *exact* quantization of the Hall conductance. It is important to note that robust topological quantum numbers have never been found in the literature on the instanton vacuum. The problem seemed somewhat lost due to the ambiguous choice of boundary conditions.[(8)] Another major problem was the electron-electron interactions which were too difficult to handle in spite of the long standing quest for a unification of all (abelian) quantum Hall states.[(9)]

2. UNIFYING RENORMALIZATION THEORY

In a series of recent papers,[(10–14)] the foundations for the much sought after unifying theory has been laid. The research proposes an effective action which essentially extends the Finkelstein theory for localization and interaction effects to include the topological effects of an instanton vacuum as well as Chern Simons gauge theory. The approach, to a large extent relies on the fact that the interacting electron gas shares many of the features that were previously found for free electrons, namely asymptotic freedom and non-perturbative renormalization, by instantons.[(10)]

A main new issue that emerged having important consequences, even for the free electron gas, is that of the massless chiral edge excitations.[(13)] It turns out that the edge of the instanton vacuum is generally gapless, and the problem can be mapped directly onto the theory of *chiral edge bosons*. From the effective action point of view, the physics of the edge appears as an exactly solvable limit of the theory in *strong coupling*.

The direct relation that exist between the chiral edge bosons and the instanton vacuum has been exploited and extended in several ways in ref. 14. In particular, by application of the Chern Simons mapping, the complete Luttinger liquid theory for abelian quantum Hall edge states is derived including the effects of disorder and Coulomb interactions, as well as the coupling of the theory to external potentials. This theory has been subsequently used to address several problems where the combined effects of smooth disorder and the Coulomb interactions give rise to somewhat unexpected results.[(13, 14)] Specifically, we mention the experimental difficulties in observing scaling in electron transport at finite temperatures,[(15)] as well as the experiments on tunneling of electrons into the fractional quantum Hall edge.[(16)]

Since the effective action can now be studied from both the weak and strong coupling side, it will provide the necessary information on the global phase structure of the quantum transport problem in the presence of random impurities. Moreover, a more recent investigation has shown that there is no Fermi liquid principle for the quantum Hall plateau transition. These results are yet to be reported.

3. TOPOLOGICAL QUANTUM NUMBERS AT LARGE *N*

In this section I briefly mention some recent results[17] on a special, limiting case of replica field theory, namely the CP^{N-1} model for large values of N.[18, 19] It is already well known that this theory is exactly solvable. The theory has several attractive features which are in many ways, similar to QCD and it has historically been a laboratory for instanton physics.

However, from the quantum Hall physics point of view, the large N theory has been dismissed from early onward for various reasons. For example, the results seem to indicate that although there is a remnant of a plateau transition at $\theta = \pi$ (or half-integer values of the Hall conductance σ_{xy}), namely a first order transition in the vacuum free energy, there are no massless excitations in the bulk that would enable the system to carry the Hall current. In addition, the topological charge of the theory seems to be unquantized.[19] This was the final blow that permanently closed the door for quantum Hall physics.

However, in the historical analyses of the theory, the subtleties of the *massless chiral edge excitations* in the problem were in fact overlooked. As was demonstrated in ref. 13, the massless edge modes do exist in the theory and they are precisely responsible for the topological charge to be unquantized. An explicit analysis that includes the edge effects has shown that the theory supports the existence of robust topological quantum numbers that would explain the precision of the quantum Hall effect.

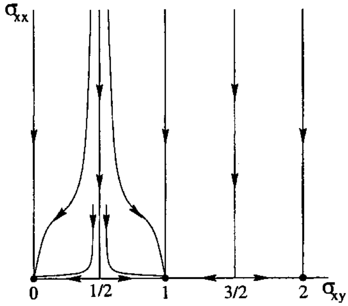

Fig. 1. Renormalization group flow diagram for the CP^{N-1} theory with large N.

The results of the theory, when evaluated in terms of the quantum Hall variables σ_{xx} and σ_{xy}, can be expressed by means of a renormalization group flow diagram as sketched in Fig. 1. There is periodicity in σ_{xy} and there are stable and unstable strong coupling fixed points at σ_{xy} equal to integer and half-integer values respectively. We have obtained explicit results for the renormalization group β functions. First, there is the known result of weak coupling ($\sigma_{xx} \gg 1$)

$$\frac{d\sigma_{xx}}{d\ln L} = -\frac{N}{2\pi} + O(\sigma_{xx})$$
$$\frac{d\sigma_{xy}}{d\ln L} = 0 \tag{1}$$

with L denoting the linear dimension of the system. Secondly, in strong coupling ($\sigma_{xx} \ll 1$) we obtain

$$\frac{d\sigma_{xx}}{d\ln L} = \sigma_{xx} \ln \frac{\sigma_{xx}}{N}$$
$$\frac{d\sigma_{xy}}{d\ln L} = 2\sigma_{xy}(1 - |\sigma_{xy}|) \ln \frac{|\sigma_{xy}|}{1 - |\sigma_{xy}|} \tag{2}$$

Here, the results hold for $|\sigma_{xy}| < \frac{1}{2}$ only. This indicates that the integer quantization $\sigma_{xy} = m$ is robust. On the other hand, for σ_{xy} close to $\frac{1}{2}$ the result can be written as

$$\frac{d\varepsilon}{d\ln L} = 2\varepsilon \tag{3}$$

where $\varepsilon = \frac{1}{2} - \sigma_{xy} \ll 1$. The eigenvalue 2 equals the dimension of the system, a well known result for first order phase transitions.

As a final remark we can say that the first order instability, along with the existence of chiral edge modes, is just sufficient to make Laughlin's gauge argument work. The large N theory is in fact a unique example of the minimal "single extended state" scenario that is needed for the quantum Hall effect to exist.

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